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An improved tip-loss correction based on vortex code results

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Abstract

Standard blade element momentum(BEM) codes use Prandtl's tip-loss correction which relies on simplified vortex theory under the assumption of optimal operating condition and no wake expansion. A new tip-loss correction for implementation in BEM codes has been developed using a lifting-line code to account for the effect of wake expansion, roll-up and distortion under any operating conditions. A database of tip-loss corrections is established for further use in BEM codes. Using this model a more physical representation of the flow and hence a better assessment of the performance of the turbine by BEM codes is expected.

Keywords: tip-loss factor, tip-loss correction, vortex methods, improved BEM, Bézier curve, wake deformation

1 Context

The origin of the most common wind turbine tip-loss correction used in BEM codes, namely Prandtl's correction, arose from the broader problem of loss minimization for lifting devices. Vortex theory predicts losses to occur for a wing of finite span and for a rotating device of finite number of blades due to the circulation flow which takes place at the tip of the lifting device. At the beginning of the 20th century, Prandtl investigated these losses for a wing(see e.g. [1]) and for a propeller blade[2]. The latter study was introduced as a simplified correction to Betz's optimal circulation[3] extending the applicability of the result from an infinite to a finite number of blades. Prandtl's model is highly simplified as it considers the axi-symmetric wake flow about a series of semi-infinite rigid lines. The following contribution from the German school was Goldstein's solution[4] of the potential flow surrounding a wake screw surface, providing a more accurate correction to Betz's circulation than Prandtl's model. Glauert[5] suggested a modification to Prandtl's tip-loss factor for a convenient numerical implementation and it is his model which has been retained to this day in most BEM

codes. It is worth mentioning that different variations of tip-loss factors are found in the literature: in [6, 7] empirical modifications are found, in [8, 9] the term "Prandtl's tip-loss factor" is used in Glauert's sense, and in [10] the result used is a special case(no tangential induction) of Glauert's factor. Though it has been Glauert's modification of Prandtl's simplified model which has been retained for BEM codes, the interest in Goldstein's factor has increased in the last decade with the development of new calculation methods[11]. Yet the generalization of the vortex theory involving Goldstein's helical screw solution for any type of loading by Theodorsen is a far-wake analysis, and the relation between far-wake parameters and rotor-parameters, though discussed for an ideal propeller[12] or wind turbine[11], has yet to be investigated for off-design or non-ideal devices. In modern practice the tip-loss factor is introduced as a ratio between the axial induction on the blade and the average axial induction[8]. Recently, refined development[13] suggests the use of an additional tip-loss factor that operates on airfoil coefficient data and accounts for three dimensional flow near the tip. A general approach to this problem is still to be found as its nature is different from the one discussed in this paper.

2 Motivations

Tip-losses are of high concern for wind energy because the load reduction they imply at the tip is associated with an important power loss. From a simple lever arm rule, the tip which is at a great distance to the hub has the possibility to generate a lot of torque. With larger rotors the tip-losses represents a significant power loss. A proper blade design can reduce these losses and increase the productivity of the blade but this requires a better understanding of tip-losses. Since most large rotors are driven primarily by load and tip deflection constraints rather than efficiency and power production constraints, a better prediction of loads near the tip and resulting tip deflection will be valuable for blade design. Moreover while BEM codes are the basis of blade design, many tip-loss functions are found in the literature[14] and

different implementations are possible[14]. Simply by varying the tip-loss factor among the different existing models, variability of power coefficient and Annual Energy Production(AEP) of $\pm 5\%$ can easily be found from one model to another[14]. This variability is not desired from a blade design perspective. Also, minor changes in the loads from one tip-loss function to the other could imply important changes in the root bending moment of the blade and have an influence on the required blade stiffness, and hence its mass and cost. This becomes critical in the development of rotors of increasing size. Therefore an improvement upon existing tip-loss corrections is needed and is developed in the following work.

3 Objectives

The objective of this study is to determine a more realistic and accurate tip-loss function than Prandtl's or Goldstein's for an implementation in BEM codes at small computational cost. Main effects found for wind turbines that are not accounted for in the analytical derivations are: expansion, roll-up and distortion of the wake. The new tip-loss function should include these effects for a more physical representation of the flow and hence a better assessment of the performance of the turbine by BEM codes.

4 Approach

Given that most of the vorticity found in the wake can be assumed to be concentrated in vortex sheets and tip-vortices, a great descriptive and predictive tool for wake dynamics is the vortex theory. Applied analytically, this theory led to the development of important theorem from Munk and Betz and the later work from Prandtl and Goldstein. In numerical applications, vortex theory gives rise to the development of different vortex codes. The implementation of an unsteady free-wake lifting-line vortex code was chosen to further study tip-losses and account for higher complexity than the earlier theoretical work[4]. The interest of such a vortex code is that it intrinsically models a finite number of blades and accounts for three-dimensional wake expansion, roll-up and distortion. The vortex code version used in this study is a lifting line code with a wake consisting of a lattice of vortex segments continuously shed in time and evolving freely. The circulation is prescribed on the lifting line and the viscous model from Squire[15] is used to mollify the singularities. Typical implementation of such code can be found in[16]. The validation of this code performed in[14] compared well with theoretical results and with similar vortex codes [17, 18].

The approach chosen was to establish a database

of tip-loss factors derived with the vortex code for a representative sample of characteristic simulations so that this database could later be used in a BEM code. For this matter, the parameters expecting to influence the tip-loss factor should be determined. The parameters selected are the circulation distribution along the blade and two "rotor state" parameters: the tip-speed ratio λ and the thrust coefficient C_T . The number of blade B has a critical importance but it is here chosen to restrict this study to three bladed wind turbines. The idea is then to determine tip-loss functions for representative sets of the selected parameters and store them in a database so that these tip-loss functions can latter be fetched directly within the BEM convergence loop for matching simulation parameters. The challenge that remains is the characterization of the different shapes of circulation curves and the discussion on this specific topic is further addressed in Sect. 5.

The vortex code can be run with a prescribed circulation, a defined tip-speed ratio and a defined geometry. The geometry dependence obviously has to be dropped for generality's sake, and a way to circumvent the fact that the thrust coefficient is dependent on the circulation has also to be found. The geometry dependence only lays on the rotor radius since the chord and twist distribution have no or limited influence in a lifting line representation[14]. An existing wind turbine geometry will be used with the rotor radius scaled to unity. All the simulation will be run with this generic rotor which makes the definition of the tip-speed ratio independent of the rotor radius. The second step is to solve the problem of interdependence between the total thrust coefficient C_T and the circulation $\Gamma(r)$. To do this, the circulation curve is normalized to unity using its maxima. The desired thrust coefficient for a given simulation will determine the multiplicative factor that should be applied to the normalized-circulation. To find this multiplicative factor prior to the vortex code simulation, a special BEM code that takes as input a prescribed circulation was implemented. This BEM code uses no drag in his formulation according to the Kutta-Joukowski formula, so that the lift coefficient is determined at each radial position as

$$C_l = \frac{2\Gamma}{V_{rel}c} \quad (1)$$

where c is the chord V_{rel} the wind velocity to the blade. An iterative procedure is used to find the multiplicative factor that should be applied to the normalized-circulation so that the BEM code returns the desired thrust coefficient for the right tip-speed ratio and normalized-circulation shape. This multiplicative factor is then used as input to the vortex code. At the end of the vortex code simulation, the thrust coefficient is computed to check if the right multiplicative constant was found. In all cases,

the two thrust coefficients matched with less than 1% of error which is within the accuracy expected on the thrust coefficient given the resolution of the database.

Using the above method, the vortex code was run for all the chosen characteristic sets of parameters. For each simulation, the tip-loss function was computed from the ratio of the azimuthally averaged axial induction in the rotor plane to its local value on the lifting line, namely:

$$F = \frac{\langle a \rangle}{a_B} \quad (2)$$

Such expression is well suited for a lifting-line formulation since the definition of the rotor plane is explicit as it indeed reduces to a plane surface or a coned-surface if the rotor is coned. To discard root-losses effect, tip-loss factors are interpolated to reach the value one at the hub. Once the database is established, it can be loaded by a new BEM code which uses the database within the BEM loop to apply iteratively the most suitable tip-loss function. The advantages of such a method is that it will provide tip-loss functions more adapted to the rotor configuration than the specific and simplified function derived by Prandtl. This tip-loss function accounts for a realistic wake geometry that expands, distorts and rolls-up.

5 Parametrization of the circulation distribution using Bézier curves

5.1 Model derivation

From the variety of shapes a circulation curve can take an explicit mathematical formalism is hard to find to parametrize all of them. From the knowledge of Bézier curves, it has been decided to develop a curve-fitting method to parametrize circulation curves using this formalism. Bézier curves are attributed to French engineer Pierre Bézier who used them for body panel design at Renault in the 1970's. Since then, they have been used by the drawing industry for e.g. fonts and vectorial drawing. The parametric Bézier curve function defined by n control points P_k in any vectorial space is

$$B(t) = \sum_{k=0}^n \frac{n!}{k!(n-k)!} t^{n-k} (1-t)^k P_k \quad (3)$$

where t element of $[0; 1]$. Given the complexity of the circulation curves, and the double change of concavity that could occur, a minimum of five control points seems to be required to model them. In a two dimensional plane (x, y) , a total of 10 parameters defining the coordinates (x_k, y_k) of the points

P_k determines the shape of the curve. An illustration of a Bézier curve with five points is plotted in Fig. 1.

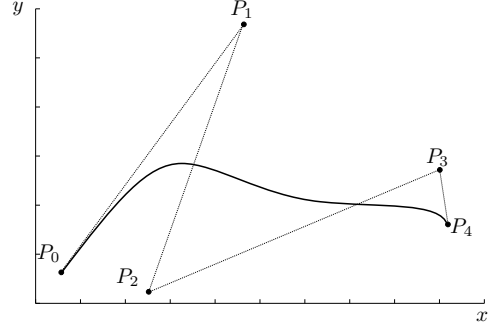


Figure 1: Bézier curve defined by five points: illustration and notations. The curve always passes through the end points and is tangent to the line between the last two and first two control points.

A further reduction of the number of parameters or unknowns is now to be performed. It is chosen to describe the circulation curve in the unit-square with the first point being located at $(0, 0)$ and the last one at $(1, 0)$. With these two assumptions four unknowns are dropped and the curve is now determined by six parameters. The slope of the tangent to the curve at any regular points, if defined, is $m(t) = y'(t)/x'(t)$. For convenience, two other parameters, t_0 and x_0 , defining the maximum of the curve are introduced to ease the reduction of parameters. They are defined such that:

$$\begin{aligned} m(t_0) &= 0 \\ x(t_0) &= x_0 \\ y(t_0) &= 1 \end{aligned} \quad (4)$$

Dummy variables $a - h$ which are purely determined by t_0 are used in the following to simplify notations. They are defined through:

$$x(t_0) = ax_1 + bx_2 + cx_3 + g \quad (5)$$

$$y(t_0) = ay_1 + by_2 + cy_3 \quad (6)$$

$$x'(t_0) = dx_1 + ex_2 + fx_3 + h \quad (7)$$

$$y'(t_0) = dy_1 + ey_2 + fy_3 \quad (8)$$

From the understanding of Bézier curves and the shapes of the circulation curves it is further assumed that the point P_3 will lay parallel to the y axis to ensure a drop of circulation at the tip, and thus x_3 is set to 1. Developing and solving Eq. (4) leads to an expression for three parameters:

$$y_2 = \frac{1}{db/a - e} \left(\frac{d}{a} + y_3 \left(f - \frac{dc}{a} \right) \right) \quad (9)$$

$$y_1 = \frac{1}{a} (1 - by_2 - cy_3) \quad (10)$$

$$x_1 = \frac{1}{a} (x_0 - bx_2 - c - g) \quad (11)$$

Table 1: Range and discretization of the different parameters used for Fig. 2

Parameter	Start	Step	End
x_2	-1	0.25	1
y_3	0	0.25	2.5
t_0	0.2	0.05	0.7

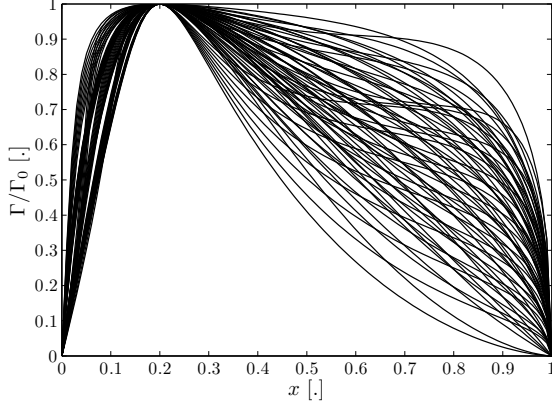


Figure 2: Example of different family of curves that can be obtained with the current parametrization. The different parameter values of Tab. 1 were used while the parameter x_0 was kept constant equal to 0.2.

The problems is now reduced to four parameters (x_0, x_2, y_3, t_0) with $x_0 \in [0; 1]$, $x_2 \in [-\infty; 1]$, $y_3 \in [0; +\infty]$, $t_0 \in [0; 1]$. Reduction constraints are added in order to reproduce realistic curves $y_1 \in [0; +\infty]$ and $x_1 \in [0; x_0 y_1]$. Other empirical constraints have been added to reduce the range of parameters sets based on the fitting of existing circulations. An example of different curve shapes that can be obtained with this parametrization is displayed in Fig. 2 for $x_0 = 0.2$. The set of parameters used for this plot can be found in Tab. 1. Out of the 1089 different combination of parameters, 63 are retained with the added constraints discussed above.

5.2 Illustration of the parametrization using existing circulation curves

In order to demonstrate the wide range of applications of the parametrization described in Sect. 5.1, different circulation curves are fitted against this model. The original circulation curves are scaled to fit in the unit-square by dividing them by their maxima Γ_0 and by reducing the radial blade span to $[0; 1]$. A least square difference criteria between the fitted and the original curves is used within a constrained optimization algorithm to determine the set of parameters best describing the original circulation. Examples of circulation curves fits are shown in Fig. 3. It can be seen that the parametrization is well suited for all different kind of circulation shapes.

Important note on the fit: As the focus is on the tip of the blade, the fitting algorithm has been set so that the curve is fitted from the maxima of circulation to the tip. Spanwise variation of circulation determines the intensity of the trailed circulation. At the maximum circulation, the trailed vorticity is zero. It is expected that the way the circulation reduces from the maxima to the tip will be the parameter influent on the tip-loss. By focusing the fit on this part of the circulation curve, more resolution is obtained, and more detailed from the original curve are reproduced by the fitted curve. Consequently, the inner part of the blade is not considered by the fitting algorithm. It is obviously possible to fit and parametrize the entire circulation curve, but at the cost of increasing the number of parameters or losing resolution at the tip of the blade.

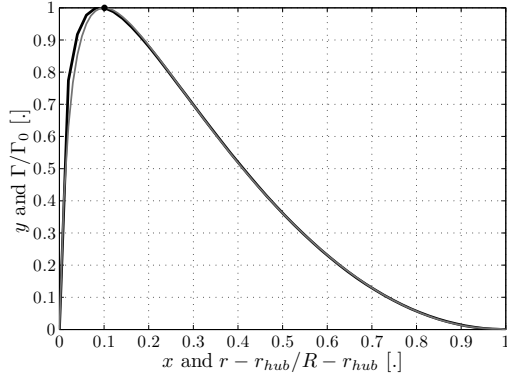
6 Sensitivity analysis and tip-loss function results

To ensure the reliability of the method described in Sect. 4 a sensitivity analysis is performed on the different indirect parameters that could potentially introduce variability on the tip-loss function. For the method to be validated it is needed that only the six parameters of choice $\{x_0, x_2, y_3, t_0, \lambda, C_T\}$ influence the tip-loss function. First, the dependence on the indirect parameters is investigated. Secondly, the relative proportion in which the chosen parameters affects the tip-loss function is studied. The influence of parameters is studied using a given prescribed circulation, with the wind speed fixed at 8m/s and the tip-speed ratio at 7.5. The design of a given wind turbine is scaled to a blade length of 1m with the chord scaled proportionally and the hub radius fixed to 0.02m.

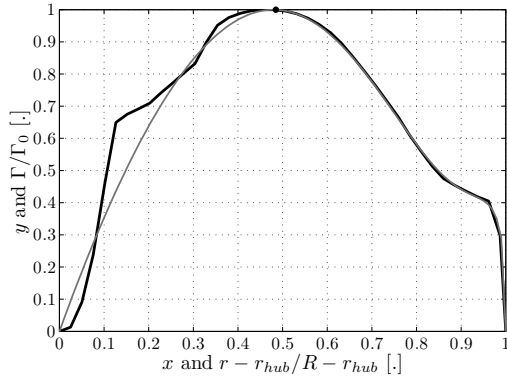
6.1 Influence of indirect parameters

The database of tip-loss correction has been established for three bladed wind turbines but can be extended to turbines with any number of blades. The dependence on the number of blades follow the same trend as the one from Glauert's correction, which is the convergence towards the function 1 as the number of blade goes to infinity. This is seen in Fig. 4.

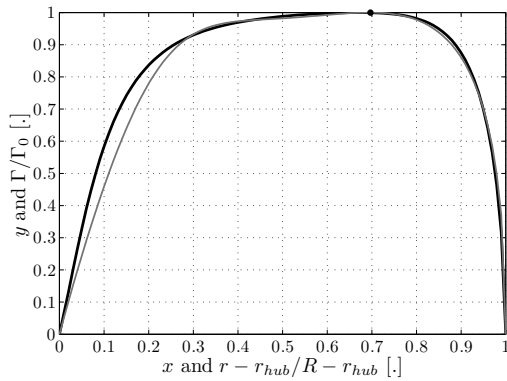
Given the lifting-line assumption for which the span-wise dimension prevails over the chord-wise and thickness-wise dimensions, it is expected that the results obtained for different chord and twist distribution will have a negligible influence on the tip-loss correction obtained with the lifting-line code.



(a)



(b)



(c)

Figure 3: Illustration of the model and fitting method developed for circulation curves. Original curves are plotted in black thick lines and fitted curve in gray thin lines. The black dots represent the maximum of the fitted curve at coordinate $(x_0, 1, t_0)$. The parametrized model developed in Sect. 5.1 is fitted to the portion of the original curve between this dot and the tip. It can be seen that the parametrization works for high (b) and low (a) lift concepts, it allows specific inflection at the tip as in (b) and fits the Goldstein circulation curves as in (c).

Such results were indeed observed while running the vortex code for different chord and twist distribution[14].

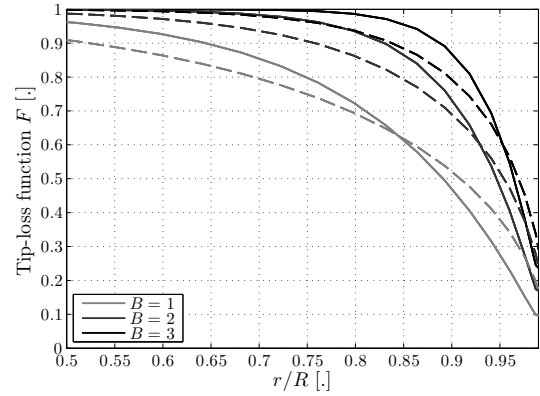


Figure 4: Dependence of the tip-loss function on the number of blades. Plain lines correspond to the tip-loss function obtained with the current method, while dashed lines are the one obtained for the same turbine configuration using Glauert's BEM-adapted method.

One of the critical parameters of vortex codes is the way the singularity is handled as a control point gets close to a vortex element. The different models presented by Leishman[19] have been tested to see the influence on the resulting tip-loss function shape. Though a dependence on the viscous model is expected, the analysis presented in Fig. 5a show that only the outermost portion of the tip-loss function is affected.

The parameters affecting the computational time are the number of spanwise segments along the blade and the rotational resolution. In Fig. 5b the influence of the number of spanwise elements is shown. A coherent convergence is observed for an increased resolution. Only small fluctuation in the resulting tip-loss function is observed when the spanwise and rotational parameters are changed.

6.2 Influence of the database parameters

For a given prescribed circulation the vortex code has been run for different turbine states determined by the couple of parameters $\{\lambda, C_T\}$. The tip-loss functions resulting in these runs can be found in Fig. 6. . In Fig. 7 five different circulation shapes have been used for a same wind turbine state, $C_T = 0.2$ and $\lambda = 5$, to study the sensitivity of the tip-loss function with respect to the circulation.

From Fig. 7 it is rather surprising to observe that a small sensitivity on the circulation distribution is observed. The distribution studied in this figure had a wide spread of bound vorticity gradient, implying different trailing vorticity, hence different mechanism of vortex emission and roll-up. From these results it seems that the importance of the parameters gov-

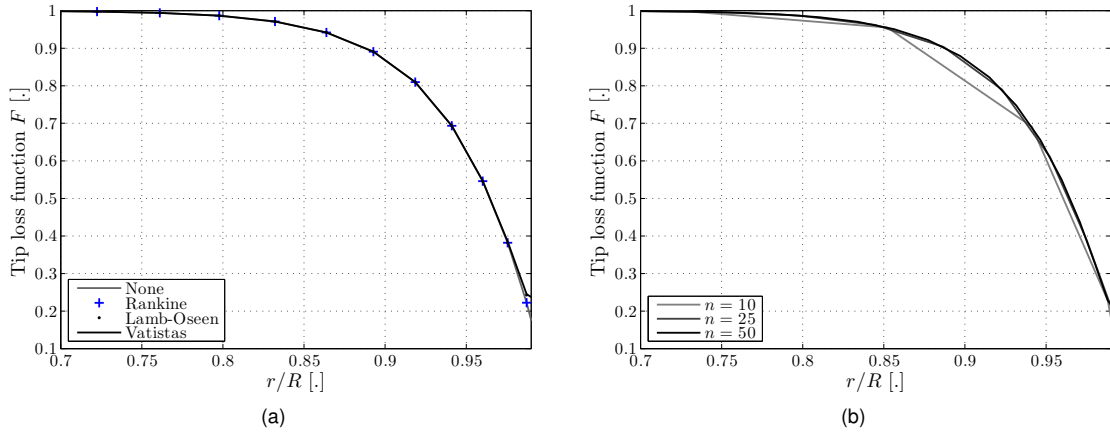


Figure 5: Sensitivity of the tip-loss function with respect to the implementation parameters. (a) Influence of the viscous model. The voutsinas model is used with $n = 2$ - (b) Influence of the grid size, n being the number of vortex rings along the blade.

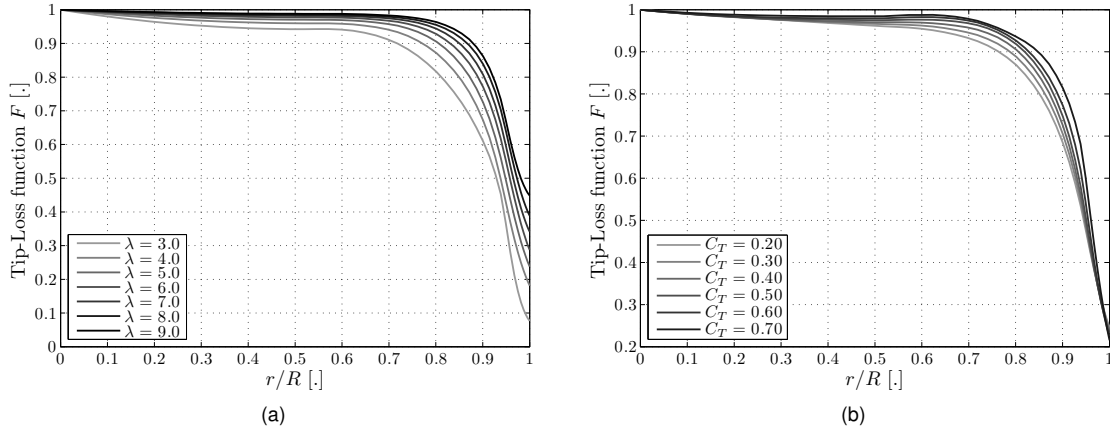


Figure 6: Influence of the turbine state $\{\lambda, C_T\}$ on the tip-loss function. (a) Influence of λ for $C_T = 0.4$ - (b) Influence of C_T for $\lambda = 5$.

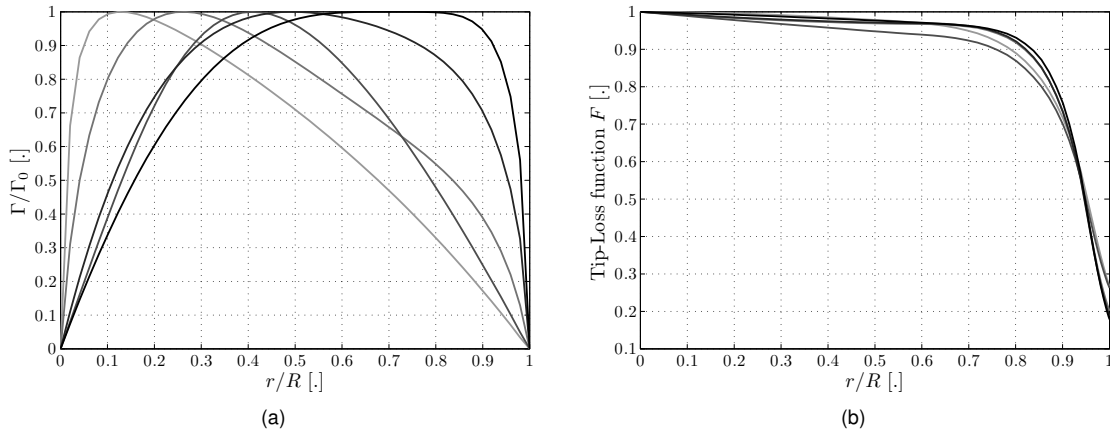


Figure 7: Influence of the circulation distribution on the tip-loss function. (a) Circulation curves - (b) Corresponding tip-loss function. The wind turbine state is $C_T = 0.2$ and $\lambda = 5$

erning the tip-loss function are in decreasing priority order, the tip-speed ratio, the thrust coefficient and the circulation shape.

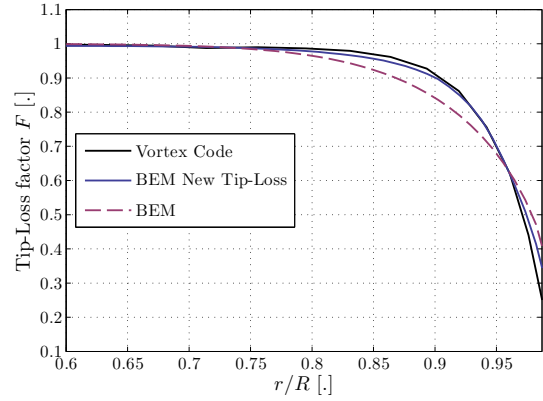
It is worth noting that an interesting aspect of the corrections obtained with the vortex code is that they don't necessarily drop to zero at the tip. It is believed by the author that this result is somehow desired: the average axial induction, though dropping to zero towards the tip, does not have to be exactly zero at the tip. The same is then expected for the tip-loss factor as defined in this context. Loads on the other hand are expected to go to zero due to the equalisation of pressure at the tip.

7 Results using the new tip-loss factor

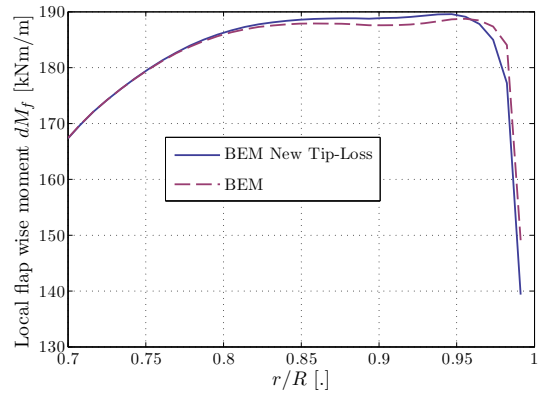
Results from a BEM code which uses an iterative vortex-database tip-loss factor were compared with simulations from a free wake vortex code and a classical BEM code that used Glauert's tip-loss factor. The new BEM code implementation[14] uses the tip-loss factor in the database whose circulation matches best the current circulation in the iterative BEM convergence loop. All three codes uses the same 2D tabulated data for the airfoil coefficients. The vortex code is hence not used with a prescribed circulation but as a predictive tool. Figure 8a illustrates one of these comparisons for a specific wind turbine simulation. The new BEM code produces results very similar to those of the vortex code and quite different at the tip than that of Glauert's BEM. It is worth noting that minor differences at the tip can have important influence on the torque or flap moments due to the large lever at this location. This can represent significant loss/gain of power and significant change in the loading at the tip which can affect the structural blade design. Similarly, computing power curves using the new BEM code and Glauert's BEM code showed differences within few percents of AEP between the two codes depending on the wind speed distributions used[14].

If it is agreed that the vortex code provides an advanced and realistic representation of the flow, then from Fig. 8 it is seen that the goal of improving the physical model of the BEM code has been reached. It should be noted that it was not obvious that the new BEM code and the vortex code when used as predictive tools would show similar results.

The overall computation time of this new BEM code is about twice the time of a normal BEM computation. During the iterative process time is spent looking up for the different circulation available and retrieving the corresponding tip-loss function from the database. A rough estimate of obtained computational time is displayed in Tab. 2, where dimensional



(a)



(b)

Figure 8: Results comparison using the different predictive tools. (a) Tip-loss factor. The new BEM code uses the new tip-loss model which differs from Glauert's BEM but matches results obtained with a vortex code. (b) Differences in the local flap moment between the two BEM codes

time unit have been used purposely for ease of representation.

Table 2: Rough comparison of typical computational times for the different codes. The new BEM code refers to the code using the new tip-loss model described in this paper.

BEM	BEM(New)	Vortex C.	CFD
1s	2s	12 min	1 day($\times 72$)

The BEM and Vortex code simulations were run on a single core machine while the Computational Fluid Dynamics(CFD) computations are usually divided between several dozens of cores(in this example 72 cores). The vortex code could also benefit from parallelization so that the computational time could be reduced. From the table it is seen that the computational time for the new BEM code is slightly increased compared to the traditional BEM code but is still small compared to a free wake vortex code or a CFD simulation. From this quick estimate it is seen

that vortex codes are too slow to be used as design tools were a lot of iterations are performed in the optimization process. On the other hand the new BEM code with the vortex-database tip-loss model obtains similar performances than the vortex code at a small computational cost giving new perspective for blade design and wind turbine aerodynamic codes.

8 Conclusions and further work

The tip-loss function derived in this study have several advantages compared to the elegant but specific and simplified functions derived by Prandtl and Goldstein. First, neither Prandtl's or Goldstein's theory account for wake expansion. Second, these theories are only valid for slightly loaded rotors operating at optimal circulation. The wake structure used is purely helical and does not include wake deformation and roll-up which are described by the vortex code. Lastly, the theories are far wake analysis that hence involve far-wake parameters which are difficult to relate to rotor parameters. On the contrary, the vortex code can provide information at the rotor disc while accounting for a wake that distorts and moves freely. The vortex code can reproduce the theoretical results of Goldstein for optimal circulation but can also be used for prediction in non-optimal situations. For these reasons, the numerical tip-loss factors obtained with such a code provides an extended and more realistic description of the flow than the one derived analytically by Prandtl and Goldstein. The use of the database tip-loss factor in an adapted BEM code proved to reproduce well the results obtained with a vortex code used as a predictive tool. The goal of improving the physical modelling of tip-losses in BEM codes has been reached if it is agreed that the vortex code provides a realistic representation of the flow. Given its small computational time the new BEM code can be used in a blade design tool and for performance prediction of wind turbines.

From the database of tip-loss corrections, a general tip-loss function could be derived as function of the parameters retained in this study. This could be done by parametrizing the tip-loss functions using for instance the same Bézier curve formalism. Using a neural network approach, a general tip-loss function matching the input database parameters and the output tip-loss parameters could be found. Such an investigation would allow a generic function of a simple form that could be directly used in a BEM code. Using vortex codes of higher complexity or a different parametrization, similar methodologies than the one presented here can be derived for further improvements on the tip-loss factor, leading to a better insight in wind turbine tip-losses.

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